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On the Matching of Transmission Cavity Stabilized Microwave Oscillators

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Abstract—**A matching condition is derived for a transmission cavity stabilized microwave oscillator, which takes account for the power loss in the diode mounting structure. In addition, the power dissipated in the damping resistor—which is commonly used in order to eliminate mode jumping problems—is minimized, thus leading to a useful improvement in both output power and loaded Q-factor of the compound oscillator structure. The effectiveness of the design procedure is finally demonstrated by applying it to a Gunn oscillator realization: at 15 GHz a loaded Q-factor of 6500 could be achieved at the sacrifice of only 2.4-dB overall power loss.**

I. Introduction

COUPLING an oscillator to a transmission cavity of a high unloaded Q-factor is well known as an efficient and simple means of improving the frequency stability. The general features involved in this method have first been discussed by Shelton [1], who introduced a damping resistor in the middle of the half-wavelength long intermediate transmission line in order to suppress unwanted modes of oscillation. The coupling line between original oscillator (diode mounting structure) and stabilizing cavity can otherwise operate as a resonator which introduces two additional potential modes of oscillation.

A theory of cavity stabilization of a microwave oscillator has been given by Ashley and Searles [2] who for the first time developed an IMPATT diode oscillator stabilized by a transmission cavity. Their theory led to very simple and efficient design formulas which can easily be applied in practice. The requirement is that the oscillator sees a matched load, which means the input reflection coefficient of the cavity has to be zero. By further taking the transmission loss of the cavity into consideration as a design objective, the input and output coupling coefficients β_1 and β_2 can be calculated. The obtainable stabilization factor is then related to the transmission loss in a simple and evident way.

The investigation of [2] leaves two problems unsolved.

1) No quantitative instruction has been given concerning the amount of damping required. Following the intentions of [2] (matching of the oscillator by putting the reflection coefficient of the cavity input to zero), one can suppose, however, that the damping resistor should present zero reflection at both ports of the intermediate transmission line.

2) It is not clear whether or not the neglect of the circuit losses of the diode mounting structure is justified.

In this present work emphasis is therefore paid to the solution of these problems.

In a recent study of Nagano and Ohnaka [3] a transmission cavity stabilized oscillator has been presented which violates the design principles of [2] in that the input impedance at the diode port of the cavity has not been matched to the characteristic impedance of the intermediate transmission line. Instead, it has been adjusted for maximum generated power. This, in our opinion, is an unnecessary and unrealistic assumption. It seems to be more adequate to suppose, for a general oscillator structure, that the active

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device is matched, i.e., that the generated power is maximized by properly choosing the physical dimensions of the diode mounting structure. Adjusting the diode mount needs not put any restriction on the magnitude of the cavity input impedance.

Furthermore, the circuit losses of the diode mounting structure (which will henceforth be called the main cavity) have been neglected in the investigations in [3]. Disposition and evaluation of the analysis of this circuit do not allow to draw generalized conclusions concerning the attainable loaded Q-factor or the necessary amount of damping. On the contrary; as the power loss in the damping resistor of the oscillator of [3] amounts to 2 dB, one may suppose that the oscillator performance is not favorable in general.

Hence, general design formulas will be derived in the following, as, e.g., a relation between the loaded Q-factor and the power loss in the stabilizing cavity (or the output power, respectively), or the minimum amount of damping, which still guarantees a single-mode operation. This is done by development and analysis of a general equivalent network.

II. Elaboration of a General Equivalent Circuit

A cavity stabilized oscillator consists of three components: the diode mounting structure, the high-Q cavity, and the coupling line with damping resistor. Each component is a resonant structure, which shall be modeled by a lumped element RLC circuit over a limited frequency range. A general equivalent network can be introduced in two ways: first, by carefully modeling the single parts of the compound structure starting from their physical meaning, and, second, in a purely formalistic manner. The latter method will be used now.

The active device is described by its admittance $y_D(\hat{v})$. In the following, admittances are normalized to the characteristic conductance G_L of the transmission line which is adjacent to the output port of the transmission cavity and forms the load of the oscillator. Normalized quantities are indicated by small letters.

For simplicity, the admittance of the active device is henceforth assumed to be real: $y_D(\hat{v}) \equiv g_D(\hat{v})$. It only depends on the RF-voltage amplitude $\hat{v}$, because the frequency dependence is weak compared with that of the passive circuitry and might thus be neglected. If the active device is driven in a parallel resonant mode, the general equivalent network of a transmission cavity stabilized oscillator is as given in Fig. 1. The H-circuit with unloaded Q-factor Q_H and loss conductance g_H represents the diode mounting structure (main cavity), the C-circuit with Q-factor Q_c and conductance g_c the stabilizing transmission cavity and load. Here g_c includes both the loss conductance of the high-Q cavity and the load conductance $g_L = 1$. Denoting the cavity input and output coupling coefficients by β_1 and β_2, respectively, one can write $Q_c = Q_0/(1 + \beta_2)$ and $g_c = 1/\beta_T = (1 + \beta_2)/\beta_1$. Q_0 means the unloaded Q-factor of the stabilizing cavity.

According to [1], the intermediate transmission line introduces a further resonant circuit which has to be damped by a damping resistor. This is modeled by the series resonant circuit of Q-factor Q_t and conductance g_{DA}. In g_{DA} two portions are combined. The first part shall be called g_t and means the inherent loss conductance which leads to the unloaded Q-factor of Q_t. The second part is the intentionally introduced amount of damping, which appears in a series connection to g_t and shall be called g'_{DA}. It is noted that Q_t is defined for $g'_{DA} \to \infty$.

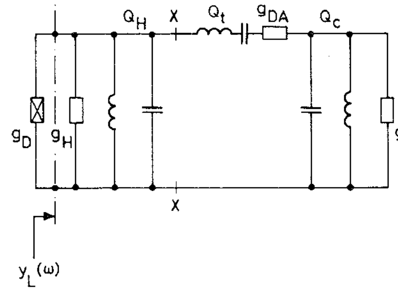

Fig. 1. Equivalent circuit for transmission cavity stabilized oscillators.
g_D Negative conductance of active device.
y_L Load admittance.
g_H Loss conductance.
g_{DA} Conductance of damping resistor.
g'_{DA} Intentionally introduced amping resistor.
g_c Cavity input conductance.
Q_H, Q_t, Q_c Q-factors.

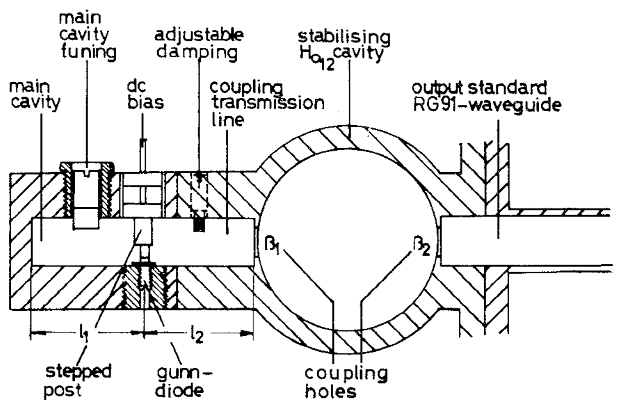

Fig. 2. Schematic cross section of the transmission cavity stabilized oscillator.

If one prefers a derivation of an equivalent network which rests upon physical considerations, one may proceed as follows. As an example, the waveguide structure, which has been used for the measurements, shall be considered. This oscillator will be described in detail later. It is schematically sketched in Fig. 2.

The Gunn element is post coupled to a rectangular waveguide. The post gives rise to some coaxial portion of the field. Thus the post may be looked at as a coaxial waveguide as seen from the active device. The characteristic impedance and electrical length of this coaxial line can be adjusted by the physical shape of the post. From this point of view the admittance levels of the active device and of the passive circuitry may be matched by means of the post which is driven to act as an impedance inverter (like, for instance, a quarter wave transformer). The exact equivalent circuit of the post is actually more complicated [4] and may be applied for detailed investigations.

As seen from the waveguide the post forms nearly a short circuit. Hence, the waveguide section to the left of the post may be modeled by a series tuned circuit at the post terminal, if its electrical length is chosen to be $\lambda_g/2$ with λ_g the guide wavelength. At the device terminal, however, the series tuned circuit is looked at through the impedance inverter as a parallel resonant circuit. Considering the waveguide section to the right of the post terminal, the stabilizing cavity is described by a series tuned circuit. In the detuned open position of the cavity the input impedance of the intermediate open ended transmission line, which has an electrical length of $\lambda_g/2$, too, appears at the post terminal. The coupling line is, hence, represented in the equivalent network by a parallel tuned circuit, which is shunted by the series tuned circuit of the stabilizing cavity. Looking again from the active device, the impedance to the right of the post has to be converted to its dual. At this reference plane, the intermediate coupling line appears as a series tuned circuit in series to the parallel tuned circuit which stands for the stabilizing cavity. Thus the equivalent network of Fig. 1 has been derived.

If the active device is driven in a series resonant mode one may proceed as done in [3]. The resulting equivalent circuit is then the dual to that of Fig. 1. The same may be achieved when the reference plane of the equivalent network is settled at the post terminal (i.e., the plane at which the post passes through the H-plane of the waveguide). Hence, the equivalent circuit of a transmission cavity stabilized oscillator, as presented in Fig. 1, can be said to be of a general validity in that it models the essential features of the compound oscillator structure.

III. Analysis of Oscillator Performance

An analysis of the circuit of Fig. 1 is set up in the admittance plane as usual. Here any operating point is given by the intersection of the "device line" $-y_D(\hat{v})$ with the "load line" $y_L(\omega)$ according to

$$y_D(\hat{v}) + y_L(\omega) = 0. \tag{1}$$

In (1) $y_L(\omega)$ means the frequency-dependent load admittance as seen by the active device. If plotted in the admittance plane (or if sketched in the Smith Chart), the load line shows one or two loops depending on the relative values of both, the unloaded Q-factors, and the natural frequencies of the various resonant circuits. As has been stated in [1], the loss conductance g_{DA} must be small enough that a loop in the load line due to the resonant circuit of the intermediate transmission line will disappear. An inspection of the equivalent network of Fig. 1 helps to formulate this statement quantitatively showing that

$$\left.\frac{db_H}{d\omega}\right|_{\omega=\omega_H} + \left.\frac{db_t}{d\omega}\right|_{\omega=\omega_t} \geq 0, \qquad \omega_H = \omega_t \tag{2}$$

must be fulfilled when the stabilizing cavity is in a detuned short position. In (2) b_H means the susceptance of the H-circuit, b_t that of the t-circuit, ω_H and ω_t being the natural frequencies of these circuits, respectively. (The detrimental consequences of a violation of $\omega_H = \omega_t$ have been discussed in [2].) Approximating the susceptances near the natural frequencies by

$$b_H = 2g_H Q_H \frac{\omega - \omega_H}{\omega_H};$$

$$b_t = -2\frac{g_t g_{DA}'^2}{(g_t + g_{DA}')^2} Q_t \frac{\omega - \omega_t}{\omega_t} \tag{3}$$

and inserting into (2) yields

$$g_{DA}' \leq \sqrt{g_t g_H Q_H / Q_t} = g_{\text{crit}}. \tag{4}$$

Here use was made of $g_t \gg g_{DA}'$. (In practice, the inherent losses of the intermediate transmission line are small compared with the added amount of damping.)

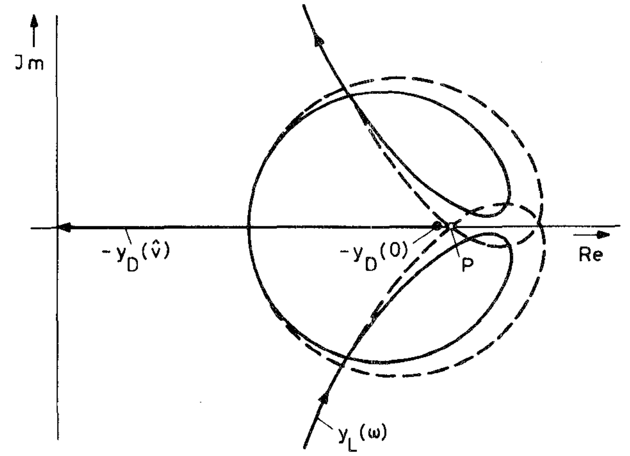

Fig. 3. Plots of load line $y_L(\omega)$ and device line $-y_D(\hat{v})$ in the admittance plane. $\hat{v}$ is RF-voltage amplitude. Solid line: admittance locus with susceptance gap. Broken line: admittance locus for minimized damping.

The influence of the damping resistor on the shape of the load line is illustrated in Fig. 3, where the natural frequency of the stabilizing cavity ω_c is tuned to $\omega_c = \omega_H = \omega_t$. When g_{DA} is chosen according to (4) ($g_{DA}' \approx g_{\text{crit}}$, solid line in Fig. 3), a small susceptance gap appears in $y_L(\omega)$. There exists, hence, a frequency margin, where only one intersection between load and device line is possible. This is at the nearly circular portion of $y_L(\omega)$, where the admittance changes are predominantly caused by the stabilizing cavity. If one wants to increase the frequency range of a single-mode operation, ordinarily the damping is increased (g_{DA} decreased). The susceptance gap then widens. This method has, e.g., been applied in [3]. As a disadvantage, the increased damping leads to a degradation of both circuit efficiency and attainable frequency stability [5].

On the other hand, the required amount of damping may be chosen in a more favorable manner. The basic idea is that a susceptance gap will not, in general, be required for a single-mode operation. The only demand is that the active device must not provide negative conductance at further potential operating points. In the example cited above, $y_L(\omega)$ may then have many intersections with the real axis of Fig. 3, but only one, where the associated real part of $y_L(\omega)$ is smaller than the small signal conductance $-g_D(0)$. Starting from the amount of damping as given from (4), a diminution of damping (increase of g'_{DA}) has a two-fold consequence: 1) the susceptance gap disappears; 2) the diameter of the loop in $y_L(\omega)$ due to the stabilizing cavity increases. Hence, $y_L(\omega)$ takes a pattern as has been schematically drawn by broken lines in Fig. 3.

$y_L(\omega)$ is restricted by two requirements.

1) The operating point on $-g_D(\hat{v})$ must give maximum power. To obtain this, $g_D(\hat{v})$ may be adjusted by the matching post (see Fig. 2).

2) The cross-over point P must lie to the left of $-g_D(0)$ in Fig. 3 to retain single-mode operation.

In order to evaluate the requirements on $y_L(\omega)$ quantitatively and thus, to determine the minimum amount of damping (maximum g'_{DA}), the RF-amplitude dependent device conductance has to be specified. For reasons of both generality and simplicity, a van der-Pol-type current voltage characteristic shall be assumed. Then the device conductance is described by

$$g_D(\hat{v}) = -g_D(0) + k\hat{v}^2 \tag{5}$$

with k being a constant. It is well known that maximum power is delivered to a load of

$$\operatorname{Re} y_L(\omega_0) = g_D(0)/2 \tag{6}$$

with

$$\hat{v}^2 = \frac{g_D(0)}{2k}. \tag{7}$$

Now the maximum value for the damping conductance (minimum damping) is calculated by setting the admittance belonging to the cross-over point P in Fig. 3 equal to the small signal conductance $-g_D(0)$, which can be replaced in terms of $y_L(\omega_0)$ by (6). This yields the maximum value of the damping conductance $g'_{DA\,\max}$

$$\frac{1}{g'_{DA\,\max}} \approx -\frac{\beta_T}{2} + \sqrt{\frac{\beta_T^2}{4} + \frac{Q_t}{g_t Q_H}\left(\frac{2}{g_H} + \beta_T\right)}, \qquad \beta_T = \frac{1}{g_c}. \tag{8}$$

Equation (8) has been derived for the detuned stabilizing cavity. The inherent insertion loss of the intermediate transmission line has again been assumed to be small (g_t large and $2Q_t/g_t Q_H \ll \beta_T$, what is fulfilled in practical oscillator circuits). $g'_{DA\,\max}$ from (8) exceeds g_{crit} from (4), thus minimizing the power loss in the damping resistor.

Provided that the intermediate transmission line is sufficiently damped according to (8), its resonant circuit might be neglected in the equivalent network of Fig. 1. Hence, the general equivalent network of a transmission cavity stabilized oscillator consists of a parallel resonant circuit in parallel to the conductance of the active device, which are both in parallel to the series connection of a conductance g_{DA} with the stabilizing parallel resonant circuit. g_{DA} should be chosen according to (8). So far, this ensures single-mode operation only for $\omega_0 = \omega_c = \omega_H$. Plotting the load line for various amounts of detuning $\omega_c \neq \omega_H$ shows, however, that the single-valued operation is preserved due to an increasing rotation and change of shape of the admittance loop with increasing detuning of the cavity.

Hence, (8) turns out to be a design formula of general validity provided that a van der Pol-type characteristic can be assumed for the active device, with Q_H and Q_t usually ranging in the same order of magnitude (≈ 1000) for practical transmission-cavity stabilized oscillators.

A. Mechanical Tuning

When the amount of damping is chosen according to (8), the consideration of the resonant character of the t-circuit may be dropped when determining the oscillation frequency. This leads to a formula for the normalized load admittance which reads

$$y_L(\omega) = g_H + g_{DA} - \frac{\dfrac{g_{DA}^2}{g_{DA}+g_c}}{1 + \left(\dfrac{g_c\Omega_c}{g_{DA}+g_c}\right)^2} + j\left[g_H\Omega_H + g_c\Omega_c\frac{\left(\dfrac{g_{DA}}{g_{DA}+g_c}\right)^2}{1+\left(\dfrac{g_c\Omega_c}{g_{DA}+g_c}\right)^2}\right]. \tag{9}$$

In (9) the normalized frequencies Ω_H and Ω_c are approximated by the linear term of a Taylor series expansion:

$$\Omega_H = 2Q_H\frac{\omega - \omega_H}{\omega_H} \qquad \Omega_c = 2Q_c\frac{\omega - \omega_c}{\omega_c}. \tag{10}$$

The oscillation frequency ω_0 is given by $\operatorname{Im} y_L(\omega_0) = 0$. Making use of

$$\left(\frac{g_c\Omega_c}{g_{DA}+g_c}\right)^2 \ll 1 \tag{11}$$

this yields

$$\omega_0 = \omega_c\frac{1 + \dfrac{g_c}{g_H}\left(\dfrac{g_{DA}}{g_{DA}+g_c}\right)^2\dfrac{Q_c}{Q_H}}{\dfrac{\omega_c}{\omega_H} + \dfrac{g_c}{g_H}\left(\dfrac{g_{DA}}{g_{DA}+g_c}\right)^2\dfrac{Q_c}{Q_H}}. \tag{12}$$

In the case of $\omega_c = \omega_H$ the oscillation frequency equals the cavity frequency. When ω_c is mechanically tuned, ω_0 will gradually differ from ω_c. The deviation $|\omega_0 - \omega_c|$ is, however, negligibly small for a large Q_c. The magnitude of the tunable frequency range depends on load line and device line as is evident from an inspection of Fig. 3. The admittance loop moves up or down when the cavity frequency is increased or decreased. The oscillator may be tuned off the midband frequency (where $\omega_0 = \omega_c = \omega_H$) as far as there is still an intersection between $-g_D(\hat{v})$ and the admittance loop. The tuning range is of course maximum when g'_{DA} is adjusted to $g'_{DA\,\max}$, because the diameter of the loop is then largest. It shall be noted that there will be no hysteresis in the tuning characteristic $\omega_0(\omega_c)$, as is, e.g., the case if the oscillator is designed according to [2].

B. Frequency Stability, Output Power, and Loaded Q

The most important feature of a cavity stabilized oscillator is its frequency stability, which is usually measured by the stabilization factor S. S is defined by the ratio of the stored energy of the stabilized oscillator to that of the unstabilized one. Hence, S can likewise be written as the ratio of the loaded Q-factors of the stabilized to the unstabilized state. Thus the stabilization factor is only a relative measure of frequency stability because it depends on the loaded Q-factor of the unstabilized oscillator. Lowering this Q-factor leads to an increase in S without improving the absolute value of the frequency stability in any way. Hence, the loaded Q-factor of the stabilized oscillator is used as an absolute measure of attained frequency stability. It can be defined via

$$Q_L = \frac{\omega_0 \left[\frac{d}{d\omega} \operatorname{Im}\, y_L(\omega)\right]_{\omega=\omega_0}}{2 \operatorname{Re}\, y_L(\omega)}. \tag{13}$$

Q_L is maximum at the midband frequency, where $\omega_0 = \omega_c = \omega_H$. Inserting $y_L(\omega_0)$ from (9) with $\omega_0 = \omega_c$ yields

$$Q_L = \frac{Q_H + Q_c \dfrac{g_c}{g_H}\left(\dfrac{g_{DA}}{g_{DA} + g_c}\right)^2}{1 + \dfrac{g_c}{g_H}\dfrac{g_{DA}}{g_{DA} + g_c}}. \tag{14}$$

The various conductances may be expressed by means of dissipated powers. Making use of (6) and (7), the maximum generated power reads

$$P_{\text{gen}} = \tfrac{1}{2} \operatorname{Re}\, y_L(\omega_0)\hat{v}^2 = \frac{1}{2k}[\operatorname{Re}\, y_L(\omega_0)]^2 \tag{15}$$

which will be used for a normalizing purpose. Defining g_x as the real part of the admittance right of the port $x - x$ in the equivalent circuit of Fig. 1, the power $p_c = P_c/P_{\text{gen}}$ dissipated in the conductance g_c reads at midband frequency

$$\begin{aligned} p_c = \frac{P_c}{P_{\text{gen}}} &= \frac{g_x}{\operatorname{Re}\, y_L(\omega_0)} \frac{g_{DA}}{g_{DA} + g_c} \\ &= g_c\left(\frac{g_{DA}}{g_{DA} + g_c}\right)^2 \frac{1}{g_H + \dfrac{g_{DA} g_c}{g_{DA} + g_c}} \end{aligned} \tag{16}$$

where Re $(y_L(\omega_0))$ has been taken from (9). The normalized output power p_{out} is defined by

$$p_{\text{out}} = \frac{\beta_2}{1 + \beta_2} \frac{P_c}{P_{\text{gen}}}. \tag{17}$$

The power losses in the cavity $p_{c,\text{loss}}$ and in the diode mounting structure $p_{H,\text{loss}}$ are given by

$$p_{c,\text{loss}} = \frac{1}{1 + \beta_2}\frac{P_c}{P_{\text{gen}}} \qquad p_{H,\text{loss}} = \frac{P_H}{P_{\text{gen}}} \tag{18}$$

with

$$\frac{P_H}{P_{\text{gen}}} = \frac{g_H}{\operatorname{Re}\, y_L(\omega_0)} = \frac{g_H}{g_H + \dfrac{g_{DA} g_c}{g_{DA} + g_c}}. \tag{19}$$

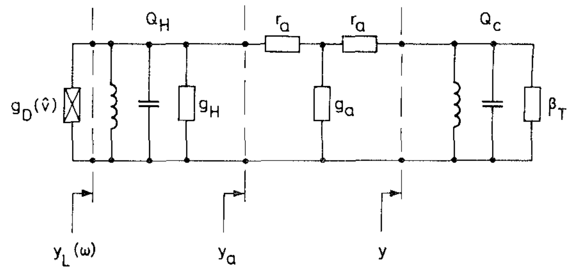

Fig. 4. Equivalent circuit for transmission cavity stabilized oscillator with attenuator

- g_D Negative conductance of active device.
- y_L Load admittance.
- y_a, y Input admittances
- g_H Loss conductance
- β_T Cavity input resistance.
- r_a, g_a Circuit elements of the attenuator.
- Q_H, Q_c Q-factors.

The various powers inserted in the equation for the loaded Q-factor yield

$$Q_L = p_H Q_H + [p_c - p_{\text{out}}]Q_0. \tag{20}$$

From (20) it can be seen that $Q_L(p_{\text{out}})$ is a straight line with slope $-Q_0$, if $\beta_T = 1/g_c = \text{const}$. This relation shows that frequency stability can be traded for output power and vice versa. It allows rapid estimation of the power losses and Q-factors required for a given loaded Q.

C. Attenuator Instead of Damping Resistor

So far, only a damping resistor has been regarded as a simple means for avoiding unwanted modes of oscillation. Realizing such a damping resistor means introducing a discontinuity in the middle of the intermediate transmission line. Following the intentions of [2], one must suppose, however, that the damping resistor should present zero reflection at both ports of the intermediate transmission line. Such a passive device will be called an attenuator in the following explanation. Using an attenuator instead of a damping resistor yields an equivalent network as presented in Fig. 4. The attenuator has been modeled by a T-section, whose input admittance $y_a(\omega_c)$ equals the load $y(\omega_c) = g_c = 1/\beta_T$. (In [2] only the special case $g_c = 1$ had been treated.)

The circuit elements g_a and r_a of the attenuator are calculated by specifying the attenuation a as the ratio of the output to the input voltage and from $y_a = 1/\beta_T$:

$$g_a = (1 - a^2)/(2a\beta_T) \qquad r_a = \beta_T(1 - a)/(1 + a). \tag{21}$$

The normalized load admittance is now

$$y_L(\omega) = g_H(1 + j\Omega_H) + \frac{1}{\beta_T}\frac{1 + \frac{1}{4}(1 - a^4)\Omega_c^2 + ja^2\Omega_c}{1 + \frac{1}{4}(1 - a^2)^2\Omega_c^2} \tag{22}$$

and yields for the maximum loaded Q-factor at midband frequency

$$Q_L = \frac{g_H Q_H + \dfrac{a^2}{\beta_T} Q_c}{\dfrac{1}{\beta_T} + g_H}. \tag{23}$$

The output power now reads

$$p_{\text{out}} = \frac{\beta_2}{1+\beta_2} \frac{\dfrac{a^2}{\beta_T}}{\dfrac{1}{\beta_T} + g_H}. \tag{24}$$

Combining (23) and (24) yields

$$Q_L = \frac{g_H Q_H}{\dfrac{1}{\beta_T} + g_H} + \left[\frac{\dfrac{a^2}{\beta_T}}{\dfrac{1}{\beta_T} + g_H} - p_{\text{out}} \right] Q_0. \tag{25}$$

This is a similar relationship between loaded Q-factor and output power as for the case of a damping resistor (20).

To ensure single-mode operation a critical attenuation a_{crit} can be defined similar to the calculation of a critical damping conductance:

$$a_{\text{crit}} = \frac{1}{\beta_T - \dfrac{1}{g_t}} \left(\sqrt{\beta_T^2 - \frac{1}{g_t^2} + \frac{Q_t}{g_t g_H Q_H}} - \sqrt{\frac{Q_t}{g_t g_H Q_H}} \right). \tag{26}$$

Here g_t and Q_t mean the loss conductance and the Q-factor, respectively, of the equivalent circuit of the intermediate transmission line.

Just as in the case of a damping resistor, the attenuation loss can be minimized. The minimum value of the attenuation $a_{\min}$ is obtained in an analogous way:

$$a_{\min}^2 = \frac{\beta_T \dfrac{g_t g_H Q_H}{Q_t} - g_H - \dfrac{1}{\beta_T}}{\beta_T \dfrac{g_t g_H Q_H}{Q_t} + g_H + \dfrac{3}{\beta_T}}. \tag{27}$$

Both (26) and (27) have been derived for the stabilizing cavity detuned. Equation (26) has been calculated with the requirement, that no additional loop due to the intermediate coupling line is present in the load line, whereas (27) implies that such a loop is allowed, but it does not introduce an additional intersection between load and device line.

IV. Discussion of Theoretical Results

With the results obtained from the analysis, an answer shall be tried to be given for the following questions.

1) What is the favorable solution for the damping of the transmission line?

2) What coupling of the stabilizing cavity will yield the best frequency stability for a given output power and a certain amount of inherent unwanted main cavity losses?

To this effect, the electrical performance of an oscillator with damping resistor will be compared to that of an oscillator with an attenuator. The comparison is made on the basis of the attainable loaded Q-factor for a given output power because the Q_L versus p_{out} relationship is thought to be the most important feature of a cavity stabilized oscillator.

The oscillators are described by their equivalent circuits of Figs. 1 and 4, respectively. In order to evaluate Q_L versus p_{out} ((20) or (25)), some circuit parameters have to be assumed: the unloaded Q-factor of the stabilizing cavity is taken as $Q_0 = 20\,000$; the unloaded Q-factor Q_H of the H-circuit can be determined from the loaded Q-factor of the unstabilized oscillator and from the loss conductance g_H. Typical values for post-coupled waveguide Gunn oscillators are $g_H = 0.1$ and $Q_H = 2000$, which are chosen here for the numerical examples. The power loss in the damping resistor and in the attenuator is minimized according to (8) and (27). In order to evaluate $g'_{DA\,\max}$ and $a_{\min}$, numerical values for Q_t and g_t have to be chosen. Assuming an intermediate transmission line made out of brass, $Q_t = 3000$ and $g_t = 100$ are thought to be quite reasonable for an oscillation frequency of 15 GHz.

The loaded Q-factor at $\omega_0 = \omega_c = \omega_H$ is plotted against the normalized output power in Fig. 5 for both an oscillator with damping resistor (solid lines) and with attenuator (broken lines). Cavity input resistance $\beta_T = 1/g_c$ and output coupling β_2 are used as parameters. An important result is that a damping resistor should be preferred as compared to an attenuator, because it leads to higher Q_L-values at the same output power. As a further result, it is seen that increasing β_2 while holding β_T constant yields higher values of output power but lower values of the loaded Q-factor. On the other hand, (for $\beta_2 =$ const.) both p_{out} and Q_L show an optimum when β_T is increased. The optima, however, lie close to one another. They are achieved at $\beta_T = 2$ (oscillator with a damping resistor) and at $\beta_T = 3$ (oscillator with an attenuator) in the examples given in Fig. 5. The optimum value of β_T is independent of β_2.

Hence, the following outlines can be stated concerning the design of transmission cavity stabilized oscillators from the results shown in Fig. 5.

1) A damping resistor instead of an attenuator should be used for achieving single-mode operation.

2) Departing from the design formulas in [2], the cavity input port in general should present a nonzero reflection coefficient ($\beta_T \neq 1$) in order to maximize the obtained loaded-Q for a required amount of output power.

The latter statements shall be illustrated by an extreme example (see Fig. 5). Utilizing an attenuator and matching the transmission cavity input port to the coupling line ($\beta_T = 1$) yields $Q_L = 2000$ at $p_{\text{out}} = 0.3$, whereas the optimum Q-factor is $Q_L = 9000$ in the case of a damping resistor and a mismatched cavity input port ($\beta_T = 2$) with the same output power. This improvement in performance is obtained by a somewhat more complicated matching procedure.

An approximate formula shall be derived for the optimum β_T value. This value is independent of the output power, as can be seen from Fig. 5. Hence, differentiating Q_L of (20) with respect to $\beta_T = 1/g_c$, while holding p_{out} constant yields

$$\beta_{T\,\text{opt}} \approx \sqrt{\frac{1}{g_{DA}} \left(\frac{1}{g_{DA}} + \frac{1}{g_H} \right)}. \tag{28}$$

In the derivation of the above expression $p_H Q_H \ll p_c Q_c$ has been implied. This assumption will be valid in most of transmission cavity stabilized oscillator configurations with

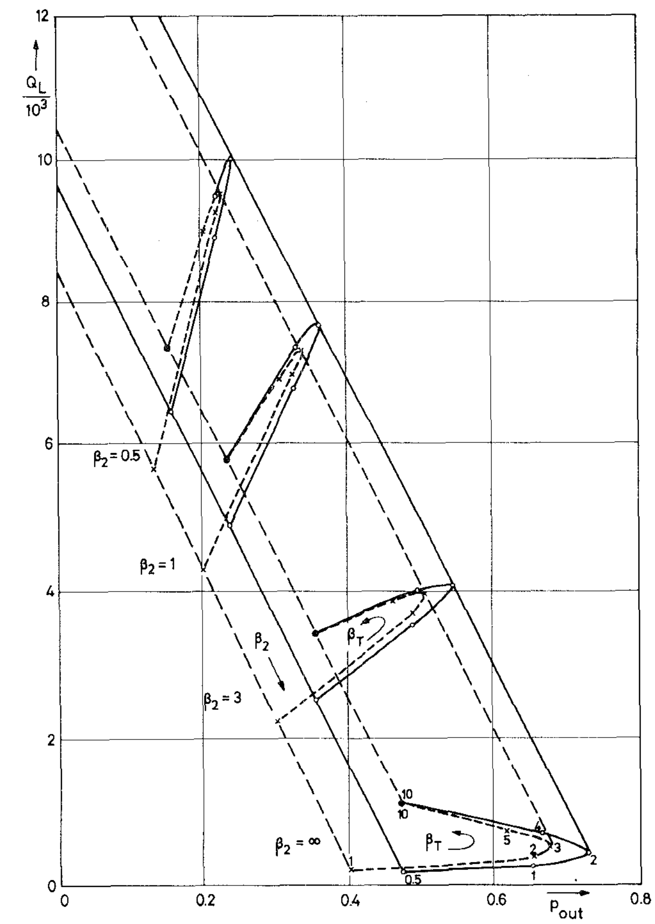

Fig. 5. Loaded Q-factor against output power of a transmission cavity stabilized oscillator. $g_H = 0.1$, $Q_H = 2000$, $g_t = 100$, $Q_t = 3000$, $Q_0 = 20\,000$. The coupling coefficients β_2, β_T increase in the direction of the arrows. Solid line: oscillator with damping resistor. Broken line: oscillator with attenuator.

high stability. In addition, the values for β_T from (28) are only approximate because g_{DA} is assumed in (20) not to depend on β_T but to be a constant. This is actually not true, as can be seen by inspection of (8). On the other hand it is not difficult to account for the β_T-dependence of g_{DA} in the differentiation of Q_L. In this case the resulting equation for $\beta_{T\,\text{opt}}$ has to be solved numerically. The validity of (28) has been checked by comparing exact and approximate β_T values for several sets of parameters (g_H, Q_H, g_t, Q_t, Q_0). Hence, (28) can be applied to design transmission cavity stabilized oscillators.

V. Experimental Results of a Transmission Cavity Stabilized Gunn Oscillator

A waveguide-type transmission cavity stabilized Gunn oscillator has been realized based on the design considerations as discussed above. A schematic cross section of the compound oscillator structure is shown in Fig. 2. The Gunn element (DGB 6839 F, Alpha Industries, output power 400 mW at 15 GHz) is post coupled to a full-height RG 91-waveguide. A stepped post has been used in order to increase the number of tuning elements for matching the active device to the load. The main cavity is formed by the short-circuited waveguide section of length l_1, which is about $\lambda_g/2$ long at the highest operating frequency. The natural frequency of the main cavity is adjusted by a tuning screw, which penetrates into the cavity from the H-plane of the waveguide. Its position is chosen halfway between post and short-circuited waveguide port. A TE_{012}-mode circular waveguide cavity is used for stabilization. Its unloaded Q-factor has been measured at 15 GHz to be 27000. The length l_2 of the intermediate transmission line has experimentally been found to be about $\lambda_g/2$. The damping resistor is located halfway between diode mount and input port of the stabilizing cavity. It consists of a tuning screw from resistive material ("EM-Airon" absorbing material, Dielectric Communications, Electronautics Department, Littleton, MA), which is mounted in a corner of the rectangular waveguide in such a way, that it travels in a direction parallel to the E-field.

In order to design a cavity stabilized oscillator the following procedure may be applied. First, the main cavity loss and Q-factor Q_H have to be determined for the unstabilized oscillator (i.e., with the stabilizing cavity removed). This may be done by standard network analyzer techniques or by the procedure proposed in [6]. In addition, Q_t and g_t of the intermediate transmission line must be specified. They can be determined through standing wave measurements with the stabilizing cavity detuned. Then g_{DA} and β_T can be calculated by solving (4) and (28) if an admittance locus with gap is desired, or from (8) and (28) if the damping resistor loss shall be minimized.

In order to compare the two potential solutions for g_{DA} and β_T, various dissipated powers have been calculated and plotted versus g_H in Fig. 6. p_{DA} means the normalized power which is dissipated in g_{DA}. Also shown is the optimum cavity input coupling β_T. Comparing the results for the admittance locus with susceptance gap (Fig. 6(a)) to those for minimized damping (Fig. 6(b)), shows that in the latter case less power is dissipated in the main cavity, whereas the power loss in the damping resistor is nearly the same for both cases. Hence, more power is available at the stabilizing cavity if the damping resistor has been minimized. This additional power may be utilized to improve the frequency stability by dissipating it in the cavity or enlarging the output power.

Continuing the design procedure the calculated β_T is then realized by the input resistance of a quarter wave transformer whose output port is connected to a matched load. This artificial load must be coupled to the main cavity with intermediate coupling line at that plane, where the input coupling hole of the stabilizing cavity will be located in the final oscillator assembly. The active device is then matched to deliver maximum power to the load. The adjustment is done by a proper selection of the physical dimensions of the post. In addition, the damping resistor is tuned to achieve single-mode operation. (The resonant frequencies of both main cavity and coupling line must of course be set such that the desired point of operation is met.)

The actual output power is usually known from the specifications, so that the amount of power which can be sacrificed in the stabilizing cavity is given from the maximum available power of a Gunn element minus the desired

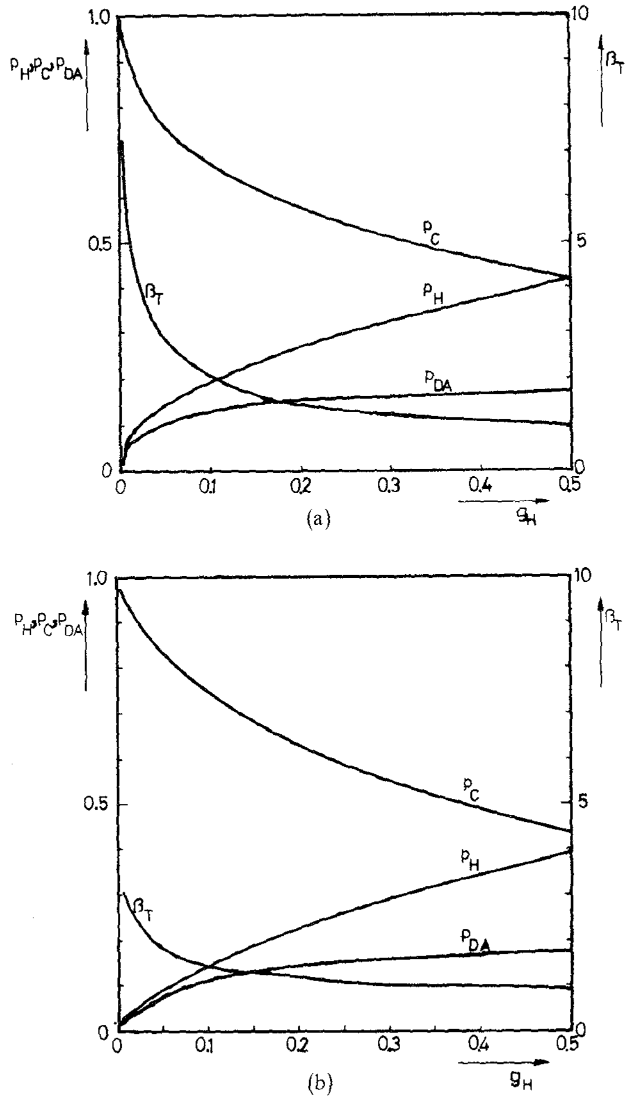

Fig. 6. Various powers of the oscillator. (a) With susceptance gap. (b) With minimized damping versus loss conductance of the main cavity.

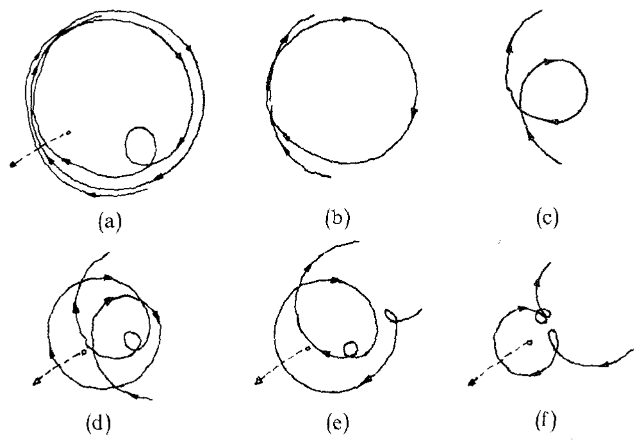

Fig. 7. Measured load line (solid) and assumed device line (broken) of a cavity stabilized Gunn oscillator. (a) Without damping. (b) Without damping, stabilizing cavity detuned. (c) With damping, stabilizing cavity detuned. (d) With damping and stabilizing cavity tuned to resonance. (e) With slightly detuned stabilizing cavity. (f) With susceptance gap.

output power. The cavity coupling coefficients β_1 and β_2 can then be calculated to meet these power requirements. The artificial load can thus be replaced by the stabilizing cavity.

The various g- and Q-parameters of the compound oscillator structure, which has been described above, have been taken as already cited above for the numerical examples of the theoretical investigations ($g_H = 0.1$, $Q_H = 2000$, $g_t = 100$, $Q_t = 3000$). The unloaded Q-factor of the stabilizing cavity of 20 000 (which is realistic for a TE_{011}-mode cavity) has only been replaced by $Q_0 = 27\,000$ for the TE_{012}-mode cavity which has been used in practice.

A trial oscillator has been realized at 15 GHz with a cavity input resistance of $\beta_t = 2$. Replacing the Gunn element by a 50-Ω coaxial line the input admittance at the diode port (i.e., the load line) has been displayed by standard network analyzer swept frequency techniques. The results are illustrated in Fig. 7, where only a section of a Smith chart is shown.

The admittance locus for the case without damping resistor and with the stabilizing cavity tuned to resonance is shown in Fig. 7(a). No single-mode operation is obviously possible. The interaction of the main cavity and the resonant intermediate transmission line become obvious when the stabilizing cavity is detuned. The corresponding load line is shown in Fig. 7(b). When a damping resistor is now provided, the diameter of the admittance loop due to the intermediate transmission line decreases. This is shown in Fig. 7(c) with a damping resistor according to (8). Tuning then the stabilizing cavity to resonance yields single-mode operation (Fig. 7(d)) even if f_c differs from $f_H = f_t$ (Fig. 7(e)). The admittance locus of Fig. 7(f) has been measured for an increased damping according to (4), what leads to a susceptance gap.

Best results were obtained with a circuit arrangement which yields the load line of Fig. 7(d). An example of the mechanical tuning characteristic for this case is given in Fig. 8. The oscillator can be tuned over a 600-MHz bandwidth without showing any hysteresis. Maximum output power is 23.6 dBm with 26 dBm available from the Gunn element. The loaded Q-factor of the oscillator amounted to $Q_L \approx 6500$. In the case of a susceptance gap according to Fig. 7(f) the output power was adjusted to remain constant. Now a Q_L of 4600 could be measured. The performance data of the oscillator with minimized damping are listed in Table I.

VI. Conclusion

Design formulas are derived for transmission cavity stabilized oscillators which take the circuit losses in the diode mounting structure and in the damping resistor into account. The formulas include a relation between the loaded Q-factor and the power loss in the stabilizing cavity (or the output power, respectively), an expression for the minimum amount of damping which still guarantees single-mode operation, and an approximate formula for the optimum input conductance of the stabilizing cavity. The theoretical results were applied to a trial cavity stabilized oscillator. At 15 GHz a loaded Q-factor of about 6500 was obtained with an overall power loss of about 2.4 dB.

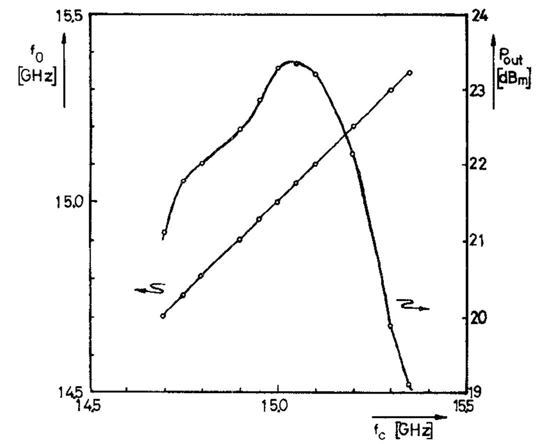

Fig. 8. Mechanical tuning characteristics.

TABLE I
PERFORMANCE DATA OF A TRANSMISSION CAVITY STABILIZED GUNN OSCILLATOR

Frequency	15 GHz
Output power	23.6 dBm
DC voltage	8 V
DC current	980 mA
Circuit efficiency ($\frac{P_{out}}{P_{gen}}$)	0.58
Loaded-Q Q_L	≈6500
Mechanically tunable bandwidth (MHz)	600 ≙ 4%
β_1	2.8
β_2	7.9
β_T	2.08
Q_0	27000
Pushing	200 $\frac{KHz}{V}$

The theory presented here exceeds that in [2], mainly by accounting for the diode mounting structure and its associated circuit losses. This leads to the tuning condition $\beta_T \neq 1$ which enhances both loaded Q-factor and output power. As stated in the investigations of [6] the circuit conductance g_H can absorb a substantial part of the output power. The circuit losses are found to range between 0.5 and 3 dB of the generated power, the higher values being valid for coaxial or MIC arrangements. Even losses of 0.5 dB can considerably influence the optimum tuning, as can be seen from the results of Fig. 5. Hence, taking the circuit losses into account leads to an improved design.

NOMENCLATURE

λ_g	Guide wavelength.
β_1	Input coupling of transmission cavity.
β_2	Output coupling of transmission cavity.
$\beta_T = \frac{1}{g_c}$	Input resistance of transmission cavity.
$\beta_{T\,\mathrm{opt}}$	Optimum input resistance of transmission cavity.
Q_H	Unloaded Q of main cavity.
Q_t	Unloaded Q of equivalent circuit of coupling line.
$Q_c = \frac{Q_0}{1+\beta_2}$	Loaded Q of transmission cavity.
Q_0	Unloaded Q of transmission cavity.
Q_L	Loaded Q of oscillator.
ω	Angular frequency.
$\omega_0 = 2\pi f_0$	Oscillation frequency.
ω_H	Resonant frequency of main cavity.
ω_t	Resonant frequency of coupling line.
$\omega_c = 2\pi f_c$	Resonant frequency of stabilizing cavity.
Ω_H	Normalized resonant frequency of main cavity.
Ω_C	Normalized resonant frequency of stabilizing cavity.
G_L	Load conductance.
g_L	Normalized load conductance.
g_T	Loss conductance of coupling line.
g'_{DA}	Intentionally introduced damping.
g_{DA}	Damping conductance.
g_{crit}	Damping conductance for impedance locus with gap.
$g'_{DA\,\max}$	Minimized damping conductance.
g_H	Loss conductance of main cavity.
g_C	Input conductance of stabilizing cavity.
g_a, r_a	Circuit elements of the attenuator.
$\hat{v}$	Voltage amplitude.
y_D	Diode admittance.
g_D	Diode conductance.
y_L	Load admittance.
b_H	Susceptance of main cavity.
b_t	Susceptance of coupling line.
a	Attenuation.
a_{crit}	Attenuation for admittance locus with gap.
$a_{\min}$	Minimum attenuation.
P_{gen}	Generated power.
p_H	Main cavity loss.
p_{DA}	Damping resistor loss.
p_c	Input power to transmission cavity.
$p_{c,\,\mathrm{loss}}$	Power loss of transmission cavity.
p_{out}	Output power.

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